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Probing states with macroscopic circulations in magnetic photonic crystals

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Abstract
We predict that when light is reflected off a magnetic photonic crystal (MPC) there is a grazing component that is *parallel* to the surface; the magnitude of this component can be changed by an external field. The direction of this parallel component is reversed as the direction of the magnetization is reversed. This provides a way to probe states with macroscopic circulations inside the MPC.

Over the last ten years there has been much activity on the Hall effect for electrons in heterostructures in which a longitudinal voltage induces a transverse component of the electrical current in the presence of an external magnetic field. This is motivated by the novel physics due to the breakdown of time reversal invariance. The Hall current is believed to be carried by edge states with macroscopic circulations. As regards the transverse motion involving electromagnetic (EM) waves, the skew scattering effect was first discussed by Rikken and co-workers [1], while the manifestation of the side jump was recently discussed by Onoda and co-workers [2].

Concurrently, there has been much activity studying the effects of resonances on the propagation of EM waves. This includes such phenomena as negative refraction, and the slowing down and the storage of light. In this paper we investigate if a scattering resonance that lacks time reversal symmetry can create novel effects on the propagation of EM waves in magnetic photonic crystals (MPCs). We predict that in the presence of an external beam on an MPC (figure 1 solid line), there is a grazing component of the reflected beam that is *parallel* to the surface of the MPC (dashed line). The magnitude of this grazing component can be changed by an external field. The direction of this parallel component is reversed (dotted line) as the direction of the magnetization is reversed. This provides for a way to probe states with macroscopic circulations inside the MPC. The effect we described is different from the Goos–Hanchen effect [2, 6] in which the mean position of a wavepacket is shifted upon reflection. In the present effect, the direction of propagation becomes parallel to the interface. We now describe our results in detail.

Our conclusion depends on the magnetic photonic band structure. To illustrate the essential physics we consider a two-dimensional (2D) MPC of a triangular array (lattice constant a) of

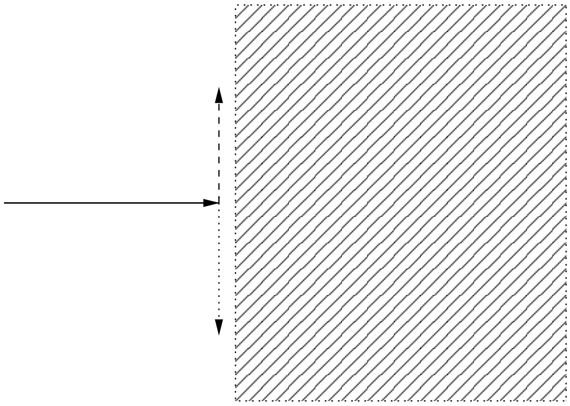

Figure 1. Figure illustrating the geometry of the effect. For an incident beam (solid line) there is a reflected beam parallel to the surface (dashed line). This grazing component will reverse direction (dotted line) when the direction of the magnetization is reversed.

magnetic cylinders with the magnetization M_0 along the axis. We expect similar effects to be manifested in MPCs of different dimensions. We first consider the scattering off a single magnetic cylinder. The magnetic susceptibility tensor and its inverse are given by [3]

$$\hat{\mu} = \begin{bmatrix} \mu & -\mathrm{i}\mu' & 0 \\ \mathrm{i}\mu' & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \hat{\mu}^{-1} = \begin{bmatrix} d & -\mathrm{i}b & 0 \\ \mathrm{i}b & d & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where μ and μ' are of a resonance form: [3]

$$\mu = 1 + \frac{\omega_m(\omega_0 + \mathrm{i}\alpha\omega)}{(\omega_0 + \mathrm{i}\alpha\omega)^2 - \omega^2}, \qquad \mu' = -\frac{\omega_m \omega}{(\omega_0 + \mathrm{i}\alpha\omega)^2 - \omega^2},$$

with a spin wave resonance frequency $\omega_0 = \gamma(H_{\text{ext}} + H_0)$ that is determined by the sum of an external field H_{ext} and the anisotropy field H_0; γ is the gyromagnetic ratio. $d = \mu/(\mu^2 - \mu'^2)$, $b = -\mu'/(\mu^2 - \mu'^2)$. The damping is controlled by a coefficient α [3]. For zero damping the diagonal element of the inverse susceptibility becomes zero ($d = 0$) at a frequency $\omega_1 = [\omega_0(\omega_0 + \omega_m)]^{1/2}$, where $\omega_m = \gamma M_0$ measures the coupling strength of the magnetic material with the EM waves. We are interested in solving Maxwell's equation,

$$(\Omega - \epsilon q_0^2)E(r) = 0, \tag{2}$$

where

$$\Omega = \nabla \times \mu^{-1} \nabla \times .$$

Here $q_0 = \omega/c$. For a single cylinder, equation (2) can be solved in cylindrical coordinates [4, 5]. There are two types of polarization, with either the electric (E mode) or the magnetic field (H mode) parallel to the cylinder axis. The wavevectors inside are $k_1 = (\epsilon/d)^{1/2} q_0$ for the E mode and $k_2 = \epsilon^{1/2} q_0$ for the H mode. By matching the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the boundary, the scattering phase shift δ_m for the mth angular momentum can be obtained. The tangent of the scattering phase shift is found to be

$$\tan \eta_n^E = \frac{J_n'(q_0 R) k_1 J - (b\, n J/(\mathrm{d}x_1) + J') J_n(q_0 R) q_0 \epsilon}{q_0 \epsilon N_n(q_0 R)(b\, n J/(\mathrm{d}x_1) + J') - k_1 J N_n'(q_0 R)} \tag{3}$$

for the E mode, with $x_1 = k_1 R$, $J = J_n(x_1)$, and $J' = J_n'(x_1)$. As can be seen, because $b \neq 0$, when the sign of n is changed, the phase shift is changed. To illustrate the essential physics, in this paper we focus here on results for polarization with the electric field along the cylinder axis.

We consider an MPC located at $x > 0$. To illustrate, we consider an incoming beam at perpendicular incidence given by $E_i = \exp(\mathrm{i}kx)$. The wavevector parallel to the interface (along y) is conserved up to a reciprocal lattice vector $\mathbf{K}_n$. The reflected beam is a sum of plane waves of different wavevectors given by $\mathbf{q}_r = (q_{nx}, K_{ny})$; $q_{nx} = \sqrt{k^2 - K_{ny}^2}$. When $K_{ny} > k$, q_{nx} is imaginary and the corresponding Fourier component is localized near the interface. For a non-magnetic photonic crystal, the amplitudes of the modes with $\mathbf{q}_r = (q_{nx}, K_{ny})$ and $\mathbf{q}'_r = (q_{nx}, -K_{ny})$ are equal. These two components together form a standing wave, and their contribution to the reflected wave is non-propagating. For an MPC, this is no longer true. The amplitudes of the two modes are not equal and the mode parallel to the interface becomes propagating.

There is another way to think of this phenomenon. Around a magnetic cylinder, the EM field can be expanded in a series of cylindrical vector basis functions M_m, N_m, with different z-component of the angular momentum m. The phase shifts for the $+|m|$ mode and the $-|m|$ modes are different and they exhibit resonances at different frequencies. When the cylinders are assembled to form an MPC, the resulting mode will look approximately like a linear combination of scattering states from each of the cylinders. Each of the functions $M_m(r)$ is proportional to $\exp(\mathrm{i}m\phi)$ and behaves like a small eddy. The sum of the eddies from each of the sites thus behaves like a giant circulation. In the study of the quantized Hall effect in electronic systems, the current is believed to be carried by edge states with macroscopic circulations. The circulating modes discussed here also exhibit macroscopic circulations. For the current system, these modes are excited by an external beam. The 'surfing' component of the reflected beam now provides for a way with which one can probe these states with macroscopic circulations.

To describe this phenomenon quantitatively, we have developed a Green's function formalism with which we can calculate the reflectivity. As far as we know, there has been no previous treatment of the reflection problem in terms of Green's functions. With this approach, one can directly relate the reflectivity to properties of the bulk system: we can relate the asymmetry of the reflected signal to the asymmetry of the Green's function of the MPC. In addition, the results are not clouded by the existence of another interface, which is always present in a finite-size numerical calculation. Following standard practice [8][3] we find that the electric field inside the MPC can be expressed in terms of its value at the surface through the integral equation

$$E(r) = -\int \mathrm{d}S \cdot \left\{ \left[\mu^{-1}\nabla \times E(r_S)\right] \times G(r, r_S)^* - \left[\mu^{-1}\nabla \times G(r, r_S)\right]^* \times E(r_S) \right\}, \tag{4}$$

where G is the Green's function of the MPC: $(\Omega' - \epsilon q_0^2)G(r, r') = \delta(r' - r)$. Here we have used units such that the speed of light $c = 1$. For the symmetry described here, at the boundary of the MPC the E field inside is the same as that in free space outside, which in turn is a sum of an incoming field $E^i \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$ and a reflected field $E^r(r)$. The derivatives of fields in the free-space side can be easily obtained from the free-space Maxwell's equations. We can calculate E^r in terms of E^i by solving equation (4). More precisely, for the reflection problem, there is translation invariance along the interface. We write the reflected wave as a Fourier series: $E^r/E^i = \sum_j e_{rj} \exp(\mathrm{i}(k_y + K_j)y - \mathrm{i}k_{xj})$, where $k_{xj} = \sqrt{\omega^2 - (k_y + K_j)^2}$. Equation (4) becomes

$$\delta_{j,0} + e_{rj} = \sum_l \mathrm{i}(k_{xl}e_{r,l} - k_x\delta_{l,0})\tilde{G}(j, l) - (\partial_x \tilde{G}(j, l))(\delta_{l,0} + e_{r,l}). \tag{5}$$

Here $\tilde{G}$ indicates the Fourier transform of G in the direction parallel to the interface.

As is explained below, we find that

$$\tilde{G}_{k_x}(r, r') = -\pi \mathrm{i} E^*_{jk_0}(r_>) E_{jk_0}(r_<)/[\epsilon(\partial_{k_y}\omega_{jk_0})\omega], \tag{6}$$

[3] We have used the fact that $\mu^*_{ab} = \mu_{ba}$ and $(\mu^{-1}\nabla \times E) \cdot \nabla \times G^* = (\nabla \times E) \cdot [\mu^{-1}\nabla \times G]^*$.

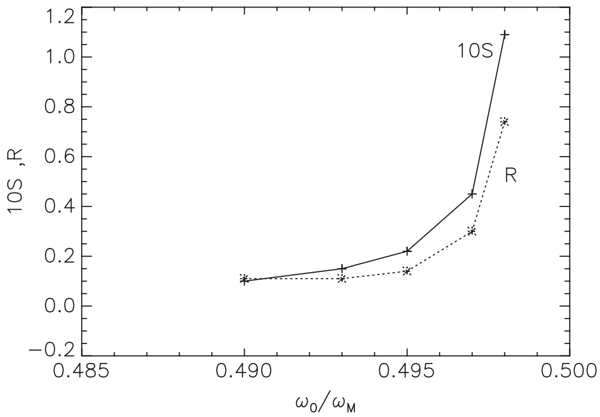

Figure 2. The tangential Poynting vector (solid line) of the reflected beam, and the reflectivity as a function of ω_0. $\omega a/(2\pi c) = 0.565$. $R/a = 0.25$, $\omega_m = 5c/a$.

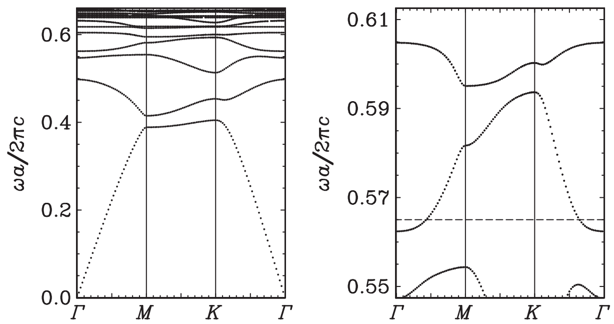

Figure 3. Photonic band structure for E modes with $R/a = 0.25$, $\omega_0 = 2.485\,c/a$, $\omega_0/\omega_m = 0.497$, $\alpha = 0$ and $\epsilon = 2.25$. The flat-band region is shown with an expanded scale on the right.

where k_0 is determined from the constraint that ω_{jk_0} is equal to the frequency of the incoming radiation. Our effect depends on the difference of the reflected beam of opposite y crystal momentum: $e_{r,j} - e_{r,-j}$. As can be seen from equation (5), this comes from the asymmetry of $\tilde{G}$, which from equation (6) in turn arises from the asymmetry of E_{jk_0}. In general, one can write the electric field as $E_{jk_0} = \sum_n f_n(r)\exp(in\phi)$. The asymmetry ultimately comes from the change of E as ϕ is changed to $-\phi$. The larger the difference $\tan\eta_n - \tan\eta_{-n}$, the larger the difference $f_n - f_{-n}$, and the stronger the surfing mode will be. This happens, for example, when a scattering resonance occurs for one of the modes.

With this technique, equation (5) can be solved and E_r/E_i is calculated; the Poynting vector of the reflected beam in the direction tangential to the interface for different values of the magnetic field (through ω_0) is evaluated. In figure 2 we show, for perpendicular incidence, the total integrated value of the Poynting vector normalized by the Poynting vector of the incoming radiation. Instead of showing a Poynting vector rapidly changing with energy, we have focused on energies close to a band gap (see figure 3, right panel) in the 'flat-band' region where the phase shift is rapidly varying (see figure 4, middle panel) and display our result in an expanded energy scale. Also shown is the reflectivity. As expected, the resulting Poynting vector is non-zero. As the band gap is approached, the reflectivity is increased, and this tangential component is increased even more. The magnitude of this component is big enough that it should be easily measurable experimentally.

To better appreciate the reflectivity we show in figure 3 the photonic band structure for the E modes for $\omega_0/\omega_m = 0.497$ along symmetry directions from $\Gamma = (0, 0)$ to $M = (0, 2\pi/\sqrt{3}a)$ to $K = (1/3, 1/\sqrt{3})2\pi/a$ and back to Γ. There are many flat bands below $\omega_1 = 0.68(2\pi c)/a$ where the phase shift changes very rapidly. A band gap in this flat-band

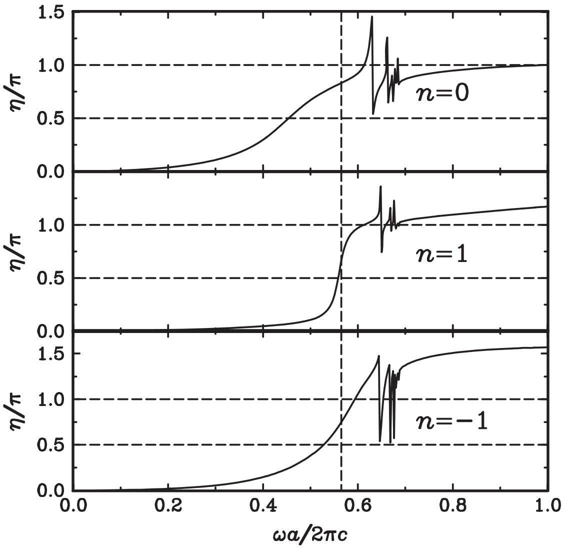

Figure 4. Phase shifts for the E modes for $n = 0$, 1 and -1 modes, with the parameters the same as in figure 3. The radius of the cylinder is $R = \frac{1}{4}(2.485\,c/\omega_0)$.

region is shown with an expanded scale in the right-hand panel. The dashed line corresponds to the frequency $0.565\,(2\pi c/a)$ that we focus on in figure 2. As ω_0 is increased, the band gap is approached and the reflectivity is increased.

The phase shifts for the E mode as a function of ω are shown in figure 4. A vertical dashed line is drawn at the frequency $0.565\,(2\pi c/a)$ that we focused on, corresponding to the horizontal dashed line in the right-hand panel of figure 3. At this frequency the phase shift η_1 for the $n = 1$ mode is near the resonance value of $\pi/2$, and is rapidly changing as a function of frequency. We thus expect states with strong macroscopic circulations to occur around there. As we explain below, rapidly varying phase shifts lead to flat bands. Note also that $\eta_{-1} \neq \eta_1$, as we expected of magnetic systems.

The equation determining the band structure is $\det[H] = 0$, where the matrix $H_{m,m'} = A(m-m') + \delta_{m,m'} \cot \eta_m$, is a function of the wavevector $\mathbf{k}$ and the circular frequency $\omega = ck_0$; A is the structure factor. Near ω_1, the change of the phase shift as the frequency is changed is given by $\cot \eta_n^H(\omega + \delta\omega) = \cot \eta_n^H(\omega) + \delta\omega X_n$, where X_n is of the order of $(\partial d/\partial\omega)/d^{3/2}$. The change of the ith eigenvalue of H can be estimated from first-order perturbation theory as $\delta\lambda_i = \langle\psi_i|X_n|\psi_i\rangle\delta\omega$ with $G|\psi_i\rangle = \lambda_i|\psi_i\rangle$. Hence, $\lambda_i + \delta\lambda_i = 0$ for $\delta\omega = -\lambda_i/X_i$. Since X_i approaches ∞ near ω_1, $\delta\omega$ approaches zero there. Thus rapidly varying phase shifts lead to flat bands.

The band structure near the flat-band region is shown in an expanded scale on the right-hand side. The frequency that we have picked corresponds to the onset of the flat-band region where we expect states with strong macroscopic circulations. Indeed, there is an increase in the surfing component as the flat bands are approached.

We next discuss the more formal aspect of the present calculation. The Hall conductivity is related to the off-diagonal part of the velocity autocorrelation function, which in turn can be expressed in terms of functions of the expectation value of the position operator $\nabla_{\mathbf{k}}$ [10]. The Poynting vector we calculated can be related to the expectation value of the velocity operator $\mathbf{\nabla_r}$, the conjugate of the position operator. To illustrate, we take the geometry of our calculation here. Then $S_y = \int \mathrm{d}y\, cE_z^* H_x/(4\pi)$. These EM fields are in the free-space side and they satisfy the free-space Maxwell's equation. In particular, we get, for our symmetry, $-\mathrm{i}\omega B_x = \nabla_y E_z$. We thus get $S_y = \int \mathrm{d}\mathbf{r}\, \mathrm{i}E_z^* \mathbf{k} \times \mathbf{\nabla_r} E_z/(|k|\mu_0 k_0 4\pi)$, as claimed. Just as time reversal invariance affects the symmetry of the position operator, the symmetry of the velocity operator is also affected. Thus even though $k_y = 0$, $S_y \neq 0$. The physical quantity discussed here does not seem to be the same as the conductivity studied in the Hall problem, however.

We close with a description of the calculation of the Green's function in equation (6). A commonly studied object in band calculations is the periodic Green's function g_k defined by $(\Omega' - \epsilon q_0^2) g_k(r, r') = \sum_R \exp(\mathrm{i}k \cdot R)\delta(r' - r - R)$ and which can be written in spectral representation in terms of the solution E_{jk} and the eigenvalue ω_{jk} of the jth photonic band as

$$g_k(r, r') = \sum_j E^*_{jk}(r) E_{jk}(r')/[\epsilon(\omega^2_{jk} - \omega^2)], \tag{7}$$

with the property that $g_k(r + R, r') = \exp(\mathrm{i}k \cdot R) g_k(r, r')$;

$$(\Omega - \epsilon q^2_{ik}) E_{ik}(r) = 0.$$

The photonic Green's function of interest is the Fourier transform of G along the interface. This Fourier transform can be related to the periodic Green's function g_k by $\tilde{G}_{kx} = \int \mathrm{d}k_y\, g_k$. This integral can be carried out with contour methods[4].

In this paper, we have focused on the reflected beam; we expect that a similar effect will also be manifested in the transmitted beam. Effects due to the non-reciprocity in magnetic photonic crystals have been discussed recently by Figotin and Vitebsky [11]. These authors assumed that the frequencies of the states at k and $-k$ are different. For the present case, the frequencies are the same. However, the average helicity of the states at k and $-k$ are different.

Acknowledgments

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[4] For a given $k_x = k_{xc}$, the poles of the integrand in equation (1) occur at a k_{yc} so that $\omega_{j,k_{xc},k_{yc}} = \pm(\omega - \mathrm{i}\epsilon)$. (The sign of ϵ is chosen so that G^* corresponds to the boundary condition of outgoing waves (see [9]).) We have an exponential factor of $\mathrm{i}k_x(x_> - x_<)$. We thus get (the pole is in the lower half of the complex plane, the contour is clockwise, and there is an additional negative sign) $G_{k_x}(r, r') = -\pi \mathrm{i} E^*_{jk_0}(r_>) E_{jk_0}(r_<)/[\epsilon(\partial_{k_y}\omega_{jk_0})\omega]$. To check, consider the one-dimensional (1D) case with constant ϵ and μ. Then $E(r) = \exp(\mathrm{i}kr)/\sqrt{2\pi}$, $k = \omega(\mu\epsilon)^{0.5}$ and we get $[G^0_k(r, r')]^* = \mathrm{i}\exp[\mathrm{i}k(r_> - r_<)]\mu/[2k]$. $G^*_{k_x}(r, r') = \pi \mathrm{i} E_{jk_0}(r_>) E^*_{jk_0}(r_<)/[\epsilon(\partial_{k_y}\omega_{jk_0})\omega]$. In the contour integral, we close the contour along path $C_\pm$ and C_∞, where along $C_\pm$, $k_y = \pm K_y/2 + \mathrm{i}p_y$; along $C_\infty k_y = \mathrm{i}\infty$. The integrand is the same along $C_\pm$ but the directions of integration are opposite. Thus contributions from C_+ and C_- cancel each other. The contribution from C_∞ is zero because of the small imaginary part.